Semester One Examination, 2022

Question/Answer booklet

MATHEMATICS METHODS UNIT 1

Section Two: Calculator-assumed

SOLUTIONS

WA student number: In figures ☐☐ ☐☐☐ ☐☐☐

In words ______________________________

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Your name ______________________________

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable): ☐

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	47	33
Section Two: Calculator-assumed	12	12	100	94	67
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed **67% (94 Marks)**

This section has **twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 8 **(6 marks)**

Four points have coordinates $A(-11,7), B(4,-3), C(7,9)$ and $D(s,t)$.

(a) If B is the midpoint of A and D, determine the value of the constant s and the value of the constant t. (2 marks)

Solution
$\frac{-11+s}{2}=4 \rightarrow s=19$ $\frac{7+t}{2}=-3 \rightarrow t=-13$
Specific behaviours
✓ value of s ✓ value of t

(b) Determine the equation of the line that is perpendicular to AB and that passes through C in the form $ax+by+c=0$, where a, b and c are integers and $a>0$. (4 marks)

Solution
$m_{AB}=\frac{-3-7}{4-(-11)}=-\frac{2}{3}, \quad m_{PERP}=-1 \div -\frac{2}{3}=\frac{3}{2}$ $y-9=\frac{3}{2}(x-7)$ $2y-18=3x-21$ $3x-2y-3=0$
Specific behaviours
✓ slope of AB ✓ slope of perpendicular line   ✓ correct equation of line, any form   ✓ correct equation of line in required form

Question 9 **(1,1,1,1,1,2,2 = 9 marks)**

The Amy car insurance company classifies its drivers according to age and gender, as shown in the following table.

		GENDER		
		Male	Female	Total
AGE	Under 25	0.15	0.12	0.27
	25 or Over	0.45	0.28	0.73
	Total	0.6	0.4	1

(a) Determine the probability that a randomly chosen driver is:

(i) Female. (1 mark)

0.4 ✓

(ii) Female and Under 25. (1 mark)

0.12 ✓

(iii) Male, or 25 or Over. (1 mark)

0.88 ✓

(iv) Female, given the driver is Under 25. (1 mark)

$$\frac{0.12}{0.27} = \frac{4}{9}$$ ✓

(v) 25 or Over, given the driver is Male. (1 mark)

$$\frac{0.45}{0.6} = \frac{3}{4}$$ ✓

For the different classes of drivers above, the probability *p,* of having at least one accident in a year, is given in the table below.

	Male	Female
Under 25	0.09	0.06
25 or Over	0.04	0.02

(b) (i) Determine the probability that a randomly chosen driver has at least one accident in a year.

$(0.09)(0.15) + (0.06)(0.12) + (0.04)(0.45) + (0.02)(0.28)$ ✓ (2 marks)

$= 0.0443$ ✓

(ii) If a driver has at least one accident in a year, what is the probability that the driver is Male and Under 25? (2 marks)

✓

$$\frac{(0.09)(0.15)}{0.0443} = 0.305$$ ✓

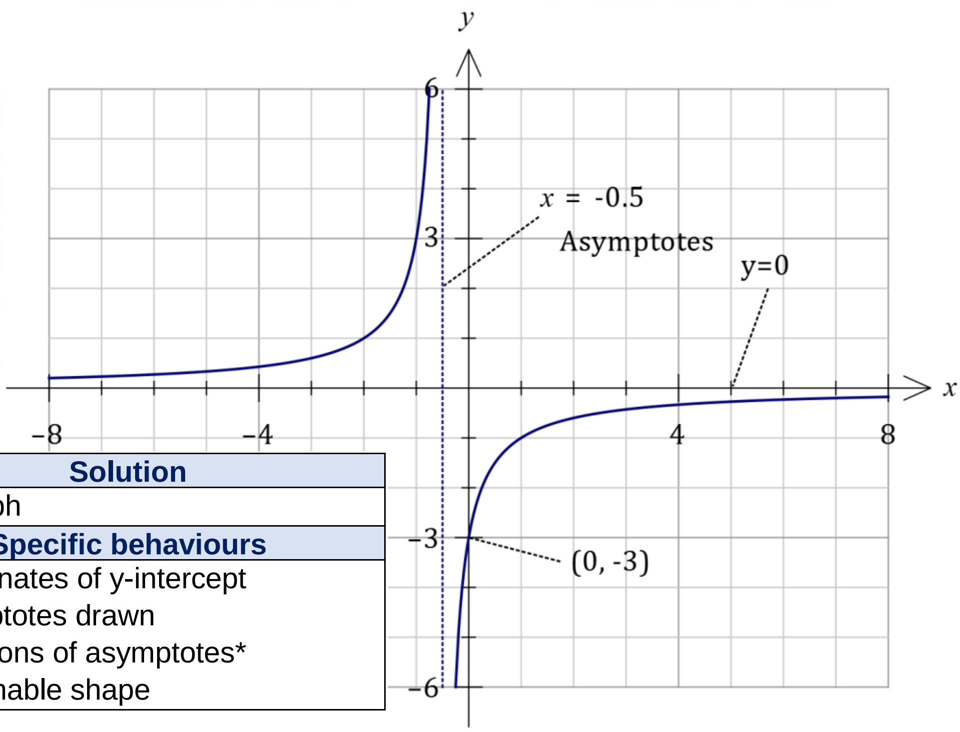
Question 10 **(9 marks)**

A function defined by $f(x) = \dfrac{a}{x+b}$, where a and b are constants, passes through the points $(-8, 0.2)$ and $(2, -0.6)$.

(a) Determine the value of a and the value of b. (3 marks)

Solution
$0.2 = \dfrac{a}{-8+b}, \quad -0.6 = \dfrac{a}{2+b}$ Solve simultaneously using CAS: $a = -1.5, b = 0.5$
Specific behaviours
✓ uses points to form two equations ✓ value of a ✓ value of b

(b) Draw the graph of $y = f(x)$ on the axes below, clearly indicating the coordinates of all axes intercepts and equations of any asymptotes. (4 marks)



Solution
See graph
Specific behaviours
✓ coordinates of y-intercept ✓ asymptotes drawn ✓ equations of asymptotes* ✓ reasonable shape

(c) State the equations of all asymptotes of the graph of $y = f(2x) - 3$. (2 marks)

Solution
$y = 0 - 3 \rightarrow y = -3, \quad x = -\dfrac{1}{2} \div 2 \rightarrow x = -\dfrac{1}{4}$
Specific behaviours
✓ equation of horizontal asymptote* ✓ equation of vertical asymptote* **Penalise only once if not equations (e.g.* HA: -3 *or* $y \neq -3$*)*

Question 11 **(8 marks)**

(a) $-\frac{a}{2} < 0$ ✓ expresses limit due to axis of symmetry (accept $\leq$ or $=$)

$a > 0$

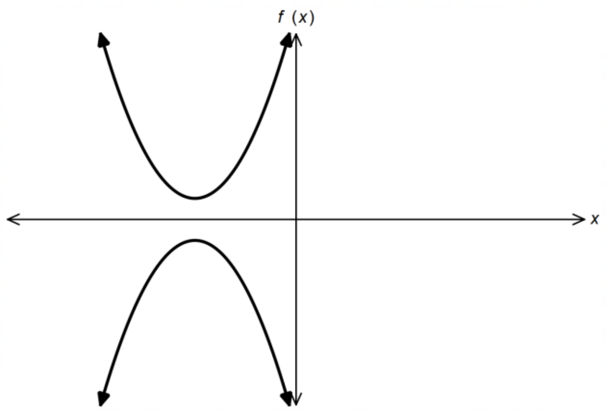
$\therefore \quad \Delta < 0 \rightarrow a^2 - 20 < 0$ ✓ expresses limit due to discriminant (accept $\leq$ or $=$)

$\therefore \quad -\sqrt{20} < a < \sqrt{20}$

$\rightarrow \quad 0 < a < \sqrt{20}$ if in the second quadrant

✓ combines limits with correct inequalities ($<$ or $>$)

(b)



✓ reasonable shape (concave down)
✓ reflected about x-axis (not TP)

(c)

$f(x)\ x^2 + 8x + 5$

$= (x+4)^2 - (4)^2 + 5$

$= (x+4)^2 - 11$

✓ completes the square
✓ states turning point (coordinates)
✓ states line of symmetry (equation)

TP: $(-4, -11)$ LOS: $x = -4$

[8]

Question 12 **(3 marks)**

A circle has equation $x^2 + y^2 + 4x - 6y = 36$

Determine the centre and radius of the circle. (3 marks)

$$(x + 2)^2 - 4 + (y - 3)^2 - 9 = 36$$

$$\therefore (x + 2)^2 + (y - 3)^2 = 49 \quad \checkmark$$

$\therefore$ Centre is (–2, 3) and radius is 7 ✓✓

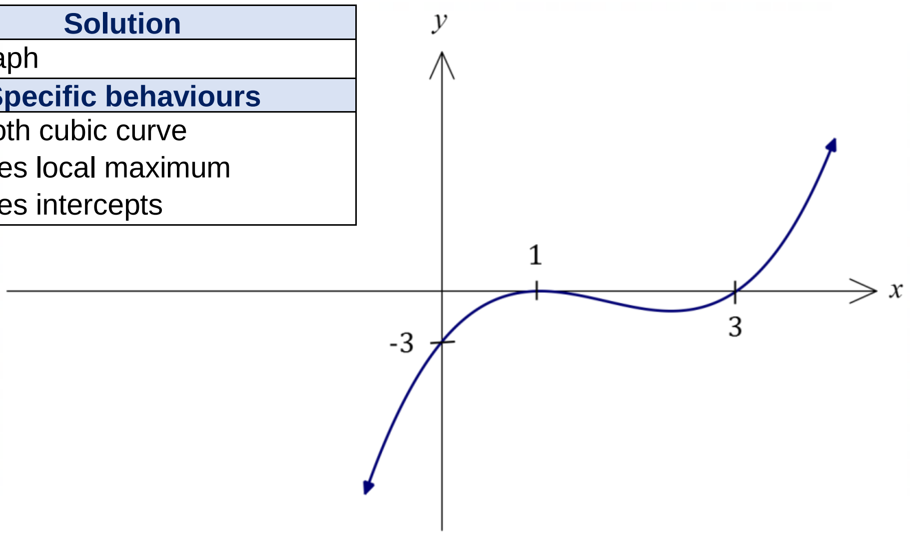
Note: 0 mark for "Centre is (-4, 6) and radius is 6".

Question 13 **(9 marks)**

The graph of the cubic polynomial $y = f(x)$ passes through the points $(3, 0)$, $(0, -3)$ and has a local maximum at $(1, 0)$.

(a) Use the above information to sketch the graph of $y = f(x)$ on the axes below. (3 marks)

Solution
See graph
Specific behaviours
✓ smooth cubic curve ✓ locates local maximum ✓ locates intercepts



Let $f(x) = x^3 + bx^2 + cx + d$, where b, c and d are constants.

(b) Determine the value of each of the constants b, c and d. (3 marks)

Solution
Factored form of cubic is $f(x) = (x-1)^2(x-3)$ Hence $f(x) = x^3 - 5x^2 + 7x - 3$ And so $b = -5,\ c = 7,\ d = -3$
Specific behaviours
✓ correct factored form of cubic ✓ expands cubic ✓ correct value for each constant

(c) Another cubic polynomial is defined by $g(x) = x^3 - 8x^2 + ax - 6$. Determine the value(s) of the constant a so that the graphs of $y = f(x)$ and $y = g(x)$ do not intersect. (3 marks)

Solution
For intersection require $f = g \to x^3 - 5x^2 + 7x - 3 = x^3 - 8x^2 + ax - 6$. Hence $3x^2 + (7-a)x + 3 = 0$. For no intersection, this quadratic must have no solution and so discriminant, $\Delta = (7-a)^2 - 4(3)(3)$, must be less than zero. Hence $(7-a)^2 < 36$ and so $1 < a < 13$.
Specific behaviours
✓ equates cubic equations and obtains quadratic ✓ uses discriminant to form inequality ✓ correct range of values for a

Question 14 **(10 marks)**

A class of 30 students are surveyed on which ATAR science subject they chose.

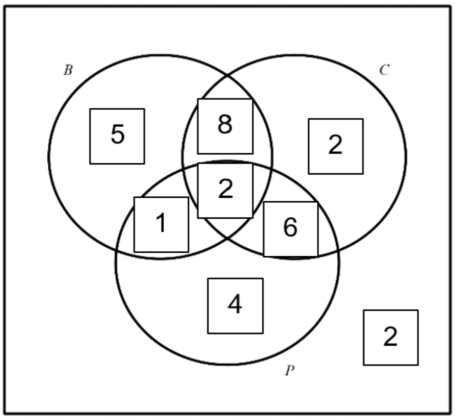
- B = Biology
- C = Chemistry
- P = Physics

Given the information:

$n(B) = 16$ $n(C) = 18$ $n(P) = 13$

$n(B \cap C) = 10$ $n(B \cap P) = 3$ $n(C \cap P \cap B') = 6$ $n(B \cap C \cap P) = 2$

(a) Use the given information to complete all regions of the Venn diagram below. (4 marks)



Marking Key
✓ Identifying $B \cap C \cap P$ ✓ Calculating the number of students who did not choose any science subjects ✓ Calculating intersection of B and C, C and P, and B and P ✓ Calculating B, C, and P OR: 2 marks if: ✓ Identifying $B \cap C \cap P$ ✓ One correct intersection of B and C, C and P, or B and P OR: 2 marks if: ✓ Identifying $B \cap C \cap P$ ✓ One correct set (B, C or P) with all 4 values correct in that set

(b) Show using set notation the set of students who chose Physics only. (1 mark)

Answer: $$P \cap (\overline{B \cup C})$$ Or: $$P \cap \overline{B} \cap \overline{C}$$

Marking Key
✓ Correct notation

(c) Show using set notation the set of students who did not choose any science subjects. (1 mark)

Answer: $$\overline{P \cup B \cup C}$$ Or: $$\overline{P} \cap \overline{B} \cap \overline{C}$$
Marking Key
✓ Correct notation

(d) A student is chosen at random from the class. Calculate:

i. $P(C|B)$ (2 marks)

Answer: $$P(C\|B) = \frac{P(C \cap B)}{P(B)} = \frac{\left(\frac{10}{30}\right)}{\left(\frac{16}{30}\right)} = \frac{5}{8}$$	Or: $$P(C\|B) = \frac{n(C \cap B)}{n(B)} = \frac{10}{16} = \frac{5}{8}$$

Marking Key
✓ Correct formula/calculation ✓ Correct answer

ii. $P\left(\overline{B \cup P}\right)$ (2 marks)

Answer: $$P\left(\overline{B \cup P}\right) = \frac{4}{30} = \frac{2}{15}$$

Marking Key
✓ Correct formula/calculation ✓ Correct answer

Question 15 **(1,2,1,2,3 = 9 marks)**

A committee of 5 is chosen to plan the ball for 2023.
If there are 10 year 11's and 5 Year 12's to choose from, determine the number of ways of selecting the committee given the following restrictions.

a) If there are no restrictions. (1 mark)

$$^{15}C_5 = \frac{15 \times 14 \times 13 \times 12 \times 11}{5 \times 4 \times 3 \times 2 \times 1}$$
$$= 3003$$

Solution
$^{15}C_5 = \frac{15 \times 14 \times 13 \times 12 \times 11}{5 \times 4 \times 3 \times 2 \times 1} = 3003$
Specific Behaviours
√ *Finds correct value*

b) It must contain 3 Year 11's and 2 Year 12's. (2 marks)

Solution
$^{10}C_3 \times {}^{5}C_2 = \frac{10 \times 9 \times 8}{3 \times 2 \times 1} \times \frac{5 \times 2}{2 \times 1} = 1200$
Specific Behaviours
√ *Shows correct calculation* √ *Gives correct value*

c) It must contain all Year 11's. (1 mark)

Solution
$^{10}C_5 = \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 1} = 252$
Specific Behaviours
√ *Finds correct solution*

d) It must contain at least 3 Year 11's. (2 marks)

Solution
$^{10}C_3 \times {}^{5}C_2 + {}^{10}C_4 \times {}^{5}C_1 + {}^{10}C_5 = 120 \times 10 + 210 \times 5 + 252 = 1200 + 1050 + 252 = 2502$
Specific Behaviours
√ *Shows correct calculation* √ *Finds correct solution*

e) It must contain at least 1 from each Year. (3 marks)

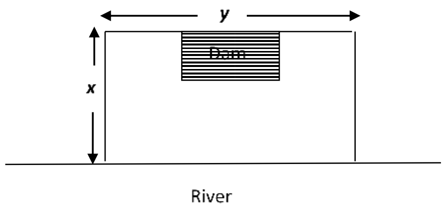
Solution
$^{10}C_4 \times {}^{5}C_1 + {}^{10}C_3 \times {}^{5}C_2 + {}^{10}C_2 \times {}^{5}C_3 + {}^{10}C_1 \times {}^{5}C_4 = 1050 + 1200 + 450 + 50 = 2750$
Specific Behaviours
√ *Shows correct calculation for at leat 2 selections* √*Shows complete correct calculation* √*Gives correct total solution*

Question 16 **(1,2,2,2 = 7 marks)**

A farmer uses 1250 metres of fencing to construct a rectangular shaped field. His land is located alongside a river, so he only needs to fence three sides of the field.

There is also a rectangular shaped dam of area 5000 m^2 within the field.

Let *x* be the width of the two shorter sides of the field.



(a) If *y* is the length of the field, express *y* in terms of *x*. Let *A* m^2 be the area of the farmable land inside the fence. (1 mark)

Solution
$2x + y = 1\,250$ $\therefore y = 1250 - 2x$
Specific behaviours
✓ writes correct equation

(b) Show that $A = -2x^2 + 1250x - 5000$ (2 marks)

Solution
$A = x(1250 - 2x) - 5000$ $= 1250x - 2x^2 - 5000$ $= -2x^2 + 1250x - 5000$
Specific behaviours
✓ Writes equation in terms of x   ✓ SHOWS development of solution

(c) Use your calculator to find the coordinates of the turning point of the graph. Give your answer to 1 decimal place. (2 marks)

Solution
coordinates of T.P. (312.5,190312.5)
Specific behaviours
✓ x value and y value   ✓ in coordinate form and to 1 decimal place

(d) Find the dimensions of the field which gives the maximum farmable area, and state the maximum area? Give your answer to 1 decimal place. (2 marks)

Solution
$x = 312.5m$ $y = 1250 - 2(312.5) = 625m$ Area = 312.5 x 625 = 195 312.5 m^2
Specific behaviours
✓ x and y values to 1 decimal place (note f.t. if penalised on previous question)   ✓ Correct Area

Question 17 **(2,2,1,2 = 7 marks)**

(a) Pascal's triangle is shown below. State the next two rows of the triangle. (2 marks)

			Row 0	1				
		Row 1	1		1			
		1		2		1		
	1		3		3		1	
1		4		6		4		1

1 5 10 10 5 1 ✓

1 6 15 20 15 6 1 ✓

(b) Solve for x using Pascal's triangle. (2 marks)

(i) $^4\mathbf{C}_x = 6$

$x = 2$ ✓

(ii) $^6\mathbf{C}_{x+1} = {}^6\mathbf{C}_{x-3}$

$x = 4$ ✓

(c) Consider the binomial expansion of $(2a - b)^7$.

(i) Which row and which term in Pascal's triangle would you use to determine the coefficient that goes with the term a^4b^3 ?

Row 7 and 4th term = 35 ✓ (1 mark)

Accept row 7 and term 3

(ii) Show that the coefficient of a^4b^3 is –560. (2 marks)

$35(2a)^4(-b)^3 = -35(16)a^4b^3$ ✓

$= -560a^4b^3$ ✓

Question 18 **(8 marks)**

(a) The graph of the quadratic function $f(x) = a(x+b)^2 + c$ has roots at $x = -1$ and $x = 9$ and the range of $f(x)$ is $y \geq -50$. Use an algebraic method to determine $f(0)$. (4 marks)

Solution
Axis of symmetry is midway between roots $(-1+9) \div 2 = 4$ and so turning point at $(4, -50)$. $$f(x) = a(x-4)^2 - 50$$ Using a root, $f(9) = 0 = a(9-4)^2 - 50 \rightarrow a = 2.$ Hence $f(0) = 2(0-4)^2 - 50 = -18.$
Specific behaviours
✓ uses range and symmetry to identify turning point or for $a(x+1)(x-9)$, $a(x-h)^2 - 50$ or $a(x-4)^4 + c$ ✓ writes equation in turning point form with constant a or CTS with $a(x+1)(x-9)$ & compares to $a(x-h)^2 - 50$ ✓ evaluates constant a ✓ states $f(0)$

(b) The area of square B is 303.5 cm^2 greater than twice the area of square A,

(i.e $B^2 = 2A^2 + 303.5$ cm^2) and the difference in the perimeters of the two squares is 50cm,

Determine the least possible area of square A, the smaller of the squares and **show all steps** of your working.

Solution
$4B = 4A + 50$ $B = A + 12.5$ Hence $$(A+12.5)^2 = 2A^2 + 303.5$$ $$A = \frac{19}{2} = 9.5, \qquad A = \frac{31}{2} = 15.5$$ Least area when $A = 9.5$: $$A_A = 9.5^2 = \frac{361}{4} = 90.25 \text{ cm}^2$$
Specific behaviours
✓ equation for difference in perimeter or states difference in sides ✓ forms quadratic equation ✓ solves quadratic equation (accept if only 9.5 given, but warn) or solves equations simultaneously ✓ states least area (do not award if TP used)

Question 19 **(9 marks)**

A chemical manufacturer has **15%** and **40%** acid solutions (i.e., 15% and 40% by volume is acid respectively) available in stock. The manufacturer needs to make up solutions from a mixture of 15% and 40% solutions. Let x be the amount of 15% acid solution required. Let y be the amount 40% acid solution required.

The manufacturer has an order for **500 litres** of a 25% acid solution.

(a) How much acid is required to produce 500 litres of 25% acid solution? (1 mark)

Solution
25% of 500 = 125 125 litres of acid are required to produce 500 litres of a 25% acid solution
Specific behaviours
✓ states correct amount of acid

(b) Determine the amount of each solution required. (4 marks)

Solution
$\left\{\begin{array}{l} 0.15x+0.4y=125 \\ x+y=500 \end{array}\right.\Big\vert_{x,\,y}$ $\{x=300, y=200\}$ 300 litres of 15% acid solution and 200 litres of 40% acid solution are required.
Specific behaviours
✓ ✓ sets up simultaneous equations correctly ✓ states correct x value ✓ states correct y value

(c) Determine x and y if the manufacturer has an order for p litres of a q% acid solution. (4 marks)

Solution
$\left\{\begin{array}{l} 0.15x+0.4y=\frac{q}{100}\times p \\ x+y=p \end{array}\right.\Big\vert_{x,\,y}$ $\left\{x=\frac{-(p\cdot q-40\cdot p)}{25}, y=\frac{p\cdot q-15\cdot p}{25}\right\}$
Specific behaviours
✓ ✓ sets up simultaneous equations correctly ✓ states correct (simplified) expression for x in terms of p & q ✓ states correct (simplified) expression for y in terms of p & q